
DDCET 2025
अब की बार - 180 पार
MATHS
INDEFINITE INTEGRATION
3
બ્રહ્માસ્ત્ર
- FREE SERIES -
- ENG + GUJ MEDIUM -

① $\int (x^3 + x^2 + 2)\,dx$

$= \frac{x^4}{4} + \frac{x^3}{3} + 2x + c$

② $\int \left(x^{\frac{3}{2}} + \frac{1}{\sqrt{x}} \right) dx$

$\int \left(x^{\frac{3}{2}} + x^{-\frac{1}{2}} \right) dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{5} x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + c$

③ $\int \left(e^{x \log_e a} + e^{a \log_e x} + e^e \right) dx$

$= \int \left(a^x + x^a + e^e \right) dx$

$= \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + e^e \cdot x + c$

④ $\int 2^x \cdot 3^x \, dx$

$\int 6^x \, dx$

$= \frac{6^x}{\log 6} + c$

$= \frac{6^x}{\log 3 + \log 2} + c$

$\underline{a}$ $\int e^{x \log_e a} \cdot e^x \, dx$

$\int a^x \cdot e^x \, dx$

$\int (ae)^x \, dx$

$= \frac{(ae)^x}{\log(ae)} + c$

$= \frac{a^x e^x}{\log a + \log_e e} + c$

$= \frac{a^x e^x}{\log a + 1} + c$

⑤ $\int \frac{3^x + 4^x}{5^x} dx$

$\int \left(\frac{3^x}{5^x} + \frac{4^x}{5^x} \right) dx$

$\int \left(\left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \right) dx$

$= \frac{\left(\frac{3}{5}\right)^x}{\log \frac{3}{5}} + \frac{\left(\frac{4}{5}\right)^x}{\log \frac{4}{5}} + c$

Q $\int \sec^2(2+3x)\,dx$

$$= \frac{\tan(2+3x)}{3} + c$$

Q $\int (2x-5)^{10}\,dx$

$$= \frac{(2x-5)^{11}}{2\times 11} + c$$

Q $\int e^{2x-3}\,dx$

$$= \frac{e^{2x-3}}{2} + c$$

Q $\int 7^{5-2x}\,dx$

$$= \frac{7^{5-2x}}{-2\cdot \log 7} + c$$

Q $\int \sin(3-x)\,dx$

$$= \frac{-\cos(3-x)}{-1} + c$$

$$= \cos(3-x) + c$$

Q $\int \cos(10x-17)\,dx$

$$= \frac{\sin(10x-17)}{10} + c$$

Q $\int \sec(2x+3)\cdot\tan(2x+3)\,dx$

$$= \frac{\sec(2x+3)}{2} + c$$

* Formula:

$(\text{Var})^{\text{con.}}$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, \; n \neq -1$$

$$\int \frac{1}{x} \, dx = \log x + c$$

$$\int e^x \, dx = e^x + c$$

$$\int a^x \, dx = \frac{a^x}{\log a} + c$$

$$\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + c$$

$\boxed{ax+b}$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \tan x \, dx = \log|\sec x|$$

$$\int \cot x \, dx = \log|\sin x|$$

$$\int \sec x \, dx = \log|\sec x + \tan x| = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$$

$$\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| = \log\left|\tan \frac{x}{2}\right|$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \tan^2 x \, dx =$$

$$\int \cot^2 x \, dx =$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx$$

$$= \tan x - x + c$$

$$\sec^2 x - \tan^2 x = 1$$

$$\sec^2 x - 1 = \tan^2 x$$

$$\int \tan^2 (3x-5)\, dx$$

$$\int \sec^2(3x-5) - 1 \quad dx$$

$$= \frac{\tan(3x-5)}{3} - x + c$$

$$\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1)\, dx$$

$$= -\cot x - x + c$$

$$\boxed{ax+b}$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos(2x)}{2}\, dx$$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$$

$$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2}\, dx$$

$$\int \sin^3 x \, dx = \int \frac{3\sin x - \sin 3x}{4}\, dx = \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$$

$$\int \cos^3 x \, dx = \int \frac{3\cos x + \cos 3x}{4}\, dx$$

Q $\displaystyle\int \frac{1}{\sqrt{1-x^1}}\,dx$

$$= \int (1-1x^1)^{-\frac{1}{2}}\,dx$$

$$= \frac{(1-x)^{\frac{1}{2}}}{-1\cdot\frac{1}{2}} + c$$

$$= -2\sqrt{1-x} + c$$

Q $f'(x) = x^2 + \dfrac{3}{x^4} \Rightarrow f(x) = \_\_\_\_\_$ and $\boxed{f(1) = 2}$; $\int f(x)\,dx$

$$\int f'(x)\,dx = \int \left(x^2 + 3x^{-4}\right) dx$$

$$f(x) = \frac{x^3}{3} + \cancel{3}\frac{x^{-3}}{-\cancel{3}} + c$$

$$\boxed{f(x) = \frac{x^3}{3} - \frac{1}{x^3} + c} \Rightarrow f(x) = \frac{x^3}{3} - \frac{1}{x^3} + \frac{8}{3}\cdot 1$$

$$= \int \left(\frac{x^3}{3} - x^{-3} + c\right) dx$$

$x = 1$

$$f(1) = \frac{1}{3} - \frac{1}{1} + c$$

$$\int f(x)\,dx = \frac{x^4}{12} - \frac{x^{-2}}{-2} + \frac{8}{3}x + K$$

$$2 = \frac{1-3}{3} + c$$

$$2 = \frac{-2}{3} + c$$

$$\frac{2}{1} + \frac{2}{3} = c$$

$$c = \frac{6+2}{3}$$

$$c = \frac{8}{3}$$

* $\int (f(x))^{n} \cdot f'(x)\, dx$

$= \dfrac{(f(x))^{n+1}}{n+1} + c$

* $\int \dfrac{f'(x)}{(f(x))^{1}}\, dx = \log|f(x)| + c$

$\int \dfrac{1}{x^2+a^2}\, dx = \dfrac{1}{a} \tan^{-1} \dfrac{x}{a} + c$

$\int \dfrac{1}{x^2-a^2}\, dx = \dfrac{1}{2a} \log\left|\dfrac{x-a}{x+a}\right| + c$

$\int \dfrac{1}{a^2-x^2}\, dx = \dfrac{1}{2a} \log\left|\dfrac{x+a}{x-a}\right| + c$

$\int \dfrac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1} \dfrac{x}{a} + c$

$\int \dfrac{1}{x\sqrt{x^2-a^2}}\, dx = \dfrac{1}{a} \sec^{-1} \dfrac{x}{a} + c$

$\int \dfrac{1}{\sqrt{x^2 \pm a^2}}\, dx = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

$\int \dfrac{1}{(2x-3)^2+5^2}\, dx$

$= \dfrac{1}{2} \cdot \dfrac{1}{5} \tan^{-1}\left(\dfrac{2x-3}{5}\right) + c$

1

If $\int f(x)dx = \frac{(\log x)^5}{5} + c$, then $f(x) = ...$

(a) $\frac{\log x}{4}$ (b) $\frac{(\log x)^5}{5}$ $\boxed{\text{(c)}}$ $\frac{(\log x)^4}{x}$ (d) $\frac{(\log x)^6}{6}$

$$\cancel{\frac{d}{dx}}\cancel{\int} f(x)\,\cancel{dx} = \frac{d}{dx}\left(\frac{(\log x)^5}{5} + c\right)$$

$$f(x) = \frac{\cancel{5}(\log x)^4}{\cancel{5}}\cdot\frac{1}{x}$$

$$\int a^x\,dx = \frac{a^x}{\log a} + c$$

2

$$\int e^{x\log_e a}\, e^x\,dx = \ldots\ldots + c$$

(a) $a^x \cdot e^x$ $\boxed{(b)}$ $\frac{(ae)^x}{(1+\log a)}$ (c) $\frac{e^x}{\log(ae)}$ (d) $\frac{a^x}{1+\log_e a}$

$$\int a^x \cdot e^x\,dx = \int (ae)^x\,dx$$

$$= \frac{(ae)^x}{\log(ae)} + c = \frac{(ae)^x}{\log a + \log_e e}$$

$$= \frac{(ae)^x}{\log a + 1}$$

$$\int (f(x))^n \cdot f'(x)\, dx$$

3

$$\int \frac{(\log x)^3}{x}\, dx = \ldots\ldots + c$$

(a) $(\log x)^2$ (b) $\frac{(\log x)^2}{2}$ (c) $\frac{1}{4}(\log x)^4$ (d) $\frac{2}{3}(\log x)^3$

$$\int (\log x)^3 \cdot \left(\frac{1}{x}\right) dx = \frac{(\log x)^4}{4} + c$$

4

$$\int sec^2 \left(5-\frac{x}{2}\right) dx = \ldots\ldots + c$$

(a) $tan\left(5-\frac{x}{2}\right)$ (b) $2tan\left(5-\frac{x}{2}\right)$ $\boxed{\text{(c)}}$ $-2tan\left(5-\frac{x}{2}\right)$ (d) $-\frac{1}{2}tan\left(5-\frac{x}{2}\right)$

$$= \frac{\tan\left(5-\frac{x}{2}\right)}{-\frac{1}{2}} + c$$

$$= -2\tan\left(5-\frac{x}{2}\right) + c$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

5

$$\int \frac{1}{4x^2+9}\,dx = \ldots\ldots + c$$

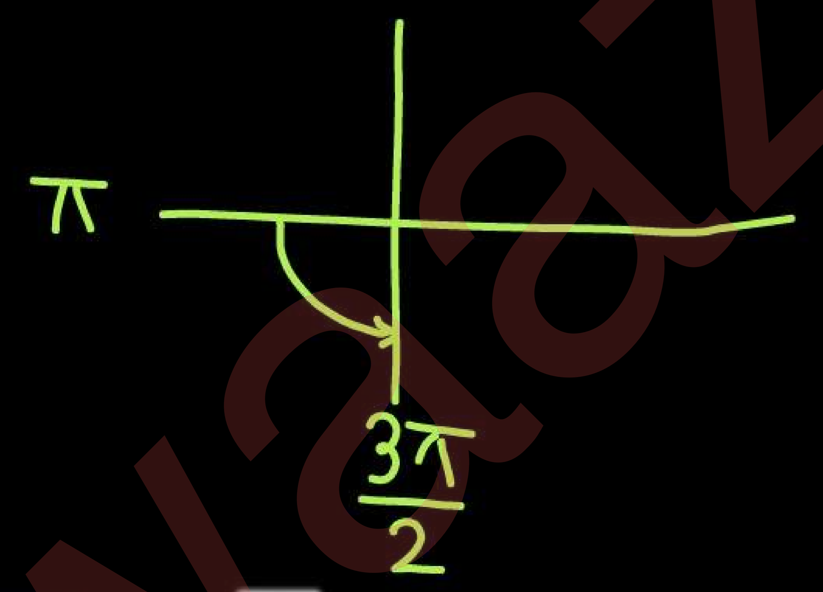
(a) $\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)$ (b) $\frac{1}{4}\tan^{-1}\left(\frac{2x}{3}\right)$ $\boxed{\text{(c)}}$ $\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)$ (d) $\frac{3}{2}\tan^{-1}\left(\frac{2x}{3}\right)$

$$\int \frac{1}{(2x)^2+3^2}\,dx = \frac{1}{2}\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)+c$$

6

$$\pi < \boxed{\frac{x}{2}} < \frac{3\pi}{2}$$

3rd



$$\int \sqrt{1-\cos x}\, dx = \ldots\ldots + c,\ \frac{2\pi}{2} < \frac{x}{2} < \frac{3\pi}{2}$$

(a) $-2\sqrt{2}\cos\frac{x}{2}$ (b) $-\sqrt{2}\cos\frac{x}{2}$ $\boxed{\text{(c)}}$ $2\sqrt{2}\cos\frac{x}{2}$ (d) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

$$\int \sqrt{2\sin^2\frac{x}{2}}\, dx = \sqrt{2}\int\left(-\sin\frac{x}{2}\right) dx$$

$$= -\sqrt{2}\left(\frac{-\cos\frac{x}{2}}{\frac{1}{2}}\right) + c$$

$$= 2\sqrt{2}\cos\frac{x}{2} + c$$

7

$$\int \frac{dx}{x\sqrt{3+\log x}} = \text{......} + c$$

$\boxed{\text{(a)}}$ $2\sqrt{3+\log x}$ (b) $\frac{2}{\sqrt{3+\log x}}$ (c) $\sqrt{3+\log x}$ (d) $-2\sqrt{3+\log x}$

$$\int (3+\log x)^{-\frac{1}{2}} \cdot \left(\frac{1}{x}\right) dx$$

$$= \frac{(3+\log x)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{3+\log x} + c$$

8

$$\int \frac{1}{\sqrt{4-3x}}\,dx = \ldots\ldots + c$$

$$= \int (4-3x)^{-\frac{1}{2}}\,dx \qquad \int x^n$$

$$= \frac{(4-3x)^{\frac{1}{2}}}{(-3)\frac{1}{2}} + c$$

$$= -\frac{2}{3}(4-3x)^{\frac{1}{2}} + c$$

(a) $-\frac{2}{3}(4-3x)^{-\frac{1}{2}} + c$

(b) $-\frac{2}{3}(4+3x)^{\frac{1}{2}}$

$\boxed{\text{(c)}}$ $-\frac{2}{3}(4-3x)^{\frac{1}{2}}$

(d) $\frac{2}{3}(4+3x)^{\frac{1}{2}}$

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

9

$$\int \frac{x-2}{x^2-4x+5}\,dx = \ldots\ldots + c$$

(a) $\log|x^2 - 4x + 5| + c$

(b) $\boxed{\text{(b)}}$ $\log\sqrt{x^2-4x+5}$

(c) $\frac{1}{2}(x^2 - 4x + 5)^2$

(d) $\log\left(\frac{x-3}{x-1}\right)$

$$\frac{1}{2}\int \frac{2x-4}{x^2-4x+5}\,dx = \frac{1}{2}\log|x^2-4x+5| + c$$

$$\log x^n = n\log x$$

$$= \log(x^2-4x+5)^{\frac{1}{2}} + c$$

$$= \log\sqrt{x^2-4x+5} + c$$

$$\int \frac{1}{x^2+a^2}\,dx$$

10

$$\int \frac{1}{3t^2+4}\,dt = \text{......} + c$$

(a) $\frac{1}{12} tan^{-1}\left(\frac{3t}{4}\right)$

(b) $\frac{1}{3}\log\left|\frac{t+2}{t-2}\right|$

$\boxed{\text{(c)}}$ $\frac{1}{2\sqrt{3}}\ tan^{-1}\left(\frac{\sqrt{3}t}{2}\right)$

(d) $\frac{1}{2\sqrt{3}}\ tan^{-1}\left(\frac{3t}{4}\right)$

$$\int \frac{1}{(\sqrt{3}t)^2+2^2}\,dt = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}t}{2}\right)+c$$

$1+\cos\theta = 2\cos^2\frac{\theta}{2}$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

11

$$\int \frac{1}{1-\cos t}\,dt = \ldots\ldots + c$$

(a) $\operatorname{cosec} t + \cot t$ $\boxed{\text{(b)}}$ $-\cot\frac{t}{2}$ (c) $-4\cot\frac{t}{2}$ (d) $\operatorname{cosec} t + \cot t$

$$\int \frac{1}{2\sin^2\frac{t}{2}}\,dt = \frac{1}{2}\int \operatorname{cosec}^2\frac{t}{2}\,dt$$

$$= \frac{\cancel{1}}{\cancel{2}}\left(\frac{-\cot\frac{t}{2}}{\frac{\cancel{1}}{\cancel{2}}}\right) + c$$

$$= -\cot\frac{t}{2} + c$$

12

$$\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}}\,dx = \ldots\ldots + c$$

(a) $e \cdot 3^{-3x}$ (b) $e^3 \log x$ $\boxed{\text{(c)}}$ $\frac{x^3}{3}$ (d) $\frac{x^2}{3}$

$$\int \frac{x^5 - x^4}{x^3 - x^2}\,dx = \int \frac{x^4\cancel{(x-1)}}{x^2\cancel{(x-1)}}\,dx$$

$$= \int x^2\,dx$$

$$= \frac{x^3}{3} + c$$

13

Good

$\int \sec^2 x \cdot \text{cosec}^2 x \; dx = \ldots\ldots + c$

(a) $\tan x + \cot x$ (b) $\tan x - \cot x$ (c) $\sec^2 x + \text{cosec}^2 x$ (d) $\cot x - \tan x$

$$\int \frac{1}{\sin^2 x \cdot \cos^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx$$

$$= \int (\sec^2 x + \text{cosec}^2 x)\, dx$$

$$= \tan x - \cot x + c$$

$* \int \sqrt{1+\sin 2x} \; dx$

$\int \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} \; dx$

$a^2 + b^2 + 2ab$

$\int \sqrt{(\sin x + \cos x)^{\cancel{2}}} \; dx$

$\int (\sin x + \cos x)\, dx = -\cos x + \sin x + c$

$$\int \frac{f'(x)}{f(x)}\,dx = \log(f(x)) + c$$

14

$\int e^{3 \log_e x} \cdot (x^4 + 1)^{-1}\, dx = \ldots\ldots + c$

(a) $\log(x^4 + 1)$ (b) $-\log(x^4 + 1)$ $\boxed{\text{(c)}}$ $\frac{1}{4}\log(x^4 + 1)$ (d) $\frac{-3}{(x^4 + 1)^2}$

$$\int \frac{x^3}{x^4+1}\,dx = \frac{1}{4}\int \frac{4x^3}{x^4+1}\,dx$$

$$= \frac{1}{4}\log|x^4+1| + c$$

15

$$\int \frac{(\log x)^4}{x} dx = \ldots\ldots + c$$

$\boxed{\text{(a)}}$ $\frac{(\log x)^5}{5}$

(b) $\frac{(\log x)^2}{2}$

(c) $\frac{\log x^5}{5x}$

(d) $\log x \cdot (\log x)^4$ -

$$\int (\log x)^4 \cdot \left(\frac{1}{x}\right) dx = \frac{(\log x)^5}{5} + c$$

16

$$\int \frac{dx}{\sqrt{1-x}} = \ldots\ldots$$

(a) $\sin^{-1}\sqrt{x} + c$ $\boxed{\text{(b)}}$ $-2\sqrt{1-x} + c$ (c) $-\sin^{-1}\sqrt{x} + c$ (d) $2\sqrt{1-x} + c$

$$\int (1-x)^{-\frac{1}{2}}\,dx = \frac{(1-1x)^{\frac{1}{2}}}{-1\cdot\frac{1}{2}} + c$$

$$= -2\sqrt{1-x} + c$$

17

$$\int \frac{(\sin x)^{99}}{(\cos x)^{101}}\, dx = \ldots\ldots + c$$

$\boxed{\text{(a)}}$ $\frac{(\tan x)^{100}}{100}$ (b) $\frac{(\tan x)^{2}}{2}$ (c) $\frac{(\tan x)^{98}}{98}$ (d) $\frac{(\tan x)^{97}}{97}$

$$\int \frac{(\sin x)^{99} \cdot 1}{(\cos x)^{99} \cos^2 x}\, dx = \int (\tan x)^{99} \cdot \sec^2 x \, dx$$

$$= \frac{(\tan x)^{100}}{100} + c$$

18

$$\int \frac{\log x^2}{x} dx = \ldots\ldots$$

(a) $\log |x^2| + c$ (b) $\log x + c$ $\boxed{\text{(c)}}$ $(\log x)^2 + c$ (d) $\frac{1}{2}(\log x)^2 + c$

$$2\int (\log x)^1 \cdot \left(\frac{1}{x}\right) dx$$

$$= 2\,\frac{(\log x)^2}{2} + c$$

$$= (\log x)^2 + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

19

$$\frac{d}{dx}\left(x\cos x - \sin x + 5\right)$$

$$- \int \frac{-x\sin x}{(x\cos x - \sin x + 5)}\,dx = \ldots\ldots + c$$

$$= 1\cdot\cancel{\cos x} - x\sin x - \cancel{\cos x} = -x\sin x$$

(a) $\log|x\cos x - \sin x + 5|$

(b) $-\log|x\cos x - \sin x + 5|$

(c) $\log|x\sin x - \cos x + 5|$

(d) $-\log|x\sin x - \cos x + 5|$

$$-\log|x\cos x - \sin x + 5|$$

20

$\int (1 - \cos x)\operatorname{cosec}^2 x \, dx = \ldots\ldots + c$

$\boxed{\text{(a)}}$ $\tan\frac{x}{2}$ (b) $\cot\frac{x}{2}$ (c) $\frac{1}{2}\tan\frac{x}{2}$ (d) $2\tan\frac{x}{2}$

$1+\cos x = 2\cos^2\frac{x}{2}$

$1-\cos x = 2\sin^2\frac{x}{2}$

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$\int \frac{1-\cos x}{\sin^2 x}dx = \int \frac{2\sin^2\frac{x}{2}}{2^2\sin^2\frac{x}{2}\cos^2\frac{x}{2}}$

$\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$

$4\sin^2\frac{x}{2}\cos^2\frac{x}{2}$

$= \frac{1}{2}\int \sec^2\frac{1x}{2}\,dx$

$= \frac{\cancel{1}\tan\frac{x}{2}}{\cancel{\frac{1}{2}}} + c$

$\int\left(\operatorname{cosec}^2 x - \frac{\cos x}{\sin x \sin x}\right)dx$

$\int(\operatorname{cosec}^2 x - \cot x \cdot \operatorname{cosec} x)\,dx$

$= -\cot x + \operatorname{cosec} x + c$

$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c$

$= \frac{1-\cos x}{\sin x} + c$

$= \frac{\cancel{2}\sin^{\cancel{2}}\frac{x}{2}}{\cancel{2}\cancel{\sin\frac{x}{2}}\cos\frac{x}{2}} = \tan\frac{x}{2} + c$

$\int \frac{\cancel{(1-\cos x)}}{(1+\cos x)\cancel{(1-\cos x)}}dx$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1^2 - \cos^2 x$

$= (1+\cos x)(1-\cos x)$

21

If $f'(x) = x^2 + 5$, then $\int f(x)dx =$ .... (c and k are arbitrary constants)

(a) $\frac{x^4}{12} + \frac{5x^2}{8} + cx + k$

(b) $-\frac{x^4}{12} - \frac{5x^2}{2} - cx + k$

(c) $\frac{x^4}{12} - \frac{5x^2}{12} + cx + k$

(d) $\frac{x^4}{12} + \frac{5x^2}{2} + cx + k$

$\int f'(x)dx = \int (x^2+5)\,dx$

$f(x) = \frac{x^3}{3} + 5x + c$

$\int f(x)dx$

$= \int \left(\frac{x^3}{3} + 5x + c\right) dx$

$= \frac{x^4}{12} + \frac{5x^2}{2} + cx + k$

22

$10^x \log 10 + 10x^9$

$$\int \frac{10x^9 + 10^x \log 10}{10^x + x^{10}}\, dx = \ldots\ldots + c$$

(a) $10^x - x^{10}$ (b) $10^x + x^{10}$ (c) $(10^x - x^{10})^{-1}$ $\boxed{\text{(d)}}$ $\log|10^x + x^{10}|$

$= \log|10^x + x^{10}| + c$

23

$\int \cos^3 x \cdot e^{\log_e \sin x}\, dx = \ldots\ldots + c$

(a) $-\dfrac{\sin^4 x}{4}$ (b) $\dfrac{e^{\sin x}}{4}$ (c) $\dfrac{e^{\cos x}}{4}$ $\boxed{\text{(d)}}$ $\dfrac{-\cos^4 x}{4}$

$-\int (\cos x)^3 \cdot (-\sin x)\, dx$

$-\dfrac{\cos^4 x}{4} + c$

$-4\,\text{sinx}$

24

$$\int \frac{\sin x}{1+4\cos x}\,dx = \ldots\ldots + c$$

(a) $\log|1+4\cos x|$

(b) $-4\log|1+4\cos x|$

$\boxed{\text{(c)}}$ $-\frac{1}{4}\log|1+4\cos x|$

(d) $-\log|1+4\cos x|$

$$\frac{1}{-4}\int \frac{-4\sin x}{1+4\cos x}\,dx$$

$$-\frac{1}{4}\log|1+4\cos x| + c$$

25

$$\int \frac{x^4 + x^2 + 1}{x^2 + 1}\,dx = \ldots\ldots\ldots + c$$

(A) $\tan^{-1} x + \frac{x^4}{4}$ $\boxed{\text{(B)}}$ $\frac{x^3}{3} + \tan^{-1} x$ (C) $\log(x^2 + 1)$ (D) $\frac{x^3}{3} + \frac{1}{2}\log\left|\frac{x+1}{x-1}\right|$

$$\int \left\{ \frac{x^2 \cancel{(x^2+1)}}{\cancel{(x^2+1)}} + \frac{1}{x^2+1} \right\} dx$$

$$\int \left(x^2 + \frac{1}{x^2+1^2}\right) dx = \frac{x^3}{3} + \frac{1}{1}\tan^{-1}\frac{x}{1} + c$$

$$* \int \underline{\sin}(\underset{\smile}{\overset{\uparrow D}{x^2}})\ x\ dx$$

$$\int e^{x^2+5 \to D}\ x\, dx$$

$$\int \frac{7^{\frac{1}{x} \to x^{-1}}}{x^2}\, dx$$

1

$$\int e^{x} \sin\left(e^{x}\right) dx = \int \sin t \; dt$$

$e^x = t$

$e^x = \frac{dt}{dx}$

$e^x dx = dt$

$= -\cos t + c$

$= -\cos(e^x) + c$

2

$$\int \cos(x^2)\, x\,dx = \int \cos t \,\frac{dt}{2}$$

D

$$x^2 = t$$

$$2x\,dx = dt$$

$$x\,dx = \frac{dt}{2}$$

$$= \frac{1}{2}\sin t + c$$

$$= \frac{1}{2}\sin(x^2) + c$$

3

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^{t}\,dt$$

D

$$= 2e^{t} + c$$

$$\sqrt{x} = t$$

$$\frac{1}{2\sqrt{x}}\,dx = dt$$

$$\frac{dx}{\sqrt{x}} = 2\,dt$$

$$= 2e^{\sqrt{x}} + c$$

I L A T E

I: $\sin^{-1}$, $\cos^{-1}$

L: $\log x$

A: 1, $\sqrt{x}$, x^2+1

T: $\cos$, $\sin$

E: 2^x, e^x

4

$$\int \cos(\underbrace{\sin x}_{D}) \cos(x)\,dx = \int \cos t \; dt$$

$$= \sin t + c$$

$$\sin x = t$$

$$= \sin(\sin x) + c$$

$$\cos x \, dx = dt$$

5

$$\int \frac{\tan\sqrt{x}\,\sec^2\sqrt{x}}{\sqrt{x}}\,dx$$

D

$= 2\int \tan t \cdot \sec^2 t \, dt$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt$

$\frac{dx}{\sqrt{x}} = 2\,dt$

$= \cancel{2}\,\frac{\tan^2 t}{\cancel{2}} + c$

$= \tan^2\sqrt{x} + c$

$2\int (\tan\sqrt{x})^1 \left(\frac{\sec^2\sqrt{x}}{2\sqrt{x}}\right) dx$

$= \cancel{2}\,\frac{\tan^2\sqrt{x}}{\cancel{2}} + c$

6

$$\int \frac{\tan(\ln x)}{x}\,dx = \int \tan t \; dt$$

$\ln x = t$

$\frac{1}{x}\,dx = dt$

$$= \log|\sec t| + c$$

$$= \log|\sec(\ln x)| + c$$

7

x

$ax+b$

p

$$\int \frac{\tan(\tan x - x)\sin^2 x}{\underbrace{1-\sin^2 x}_{\cos^2 x}}\,dx = \int \tan(\tan x - x)\cdot \tan^2 x \, dx$$

$\tan x - x = t$

$(\sec^2 x - 1)\,dx = dt$

$\tan^2 x \, dx = dt$

$$\int \tan t \, dt$$

$$= \log|\sec t| + c$$

$$= \log|\sec(\tan x - x)| + c$$

8

Good

$$\int \frac{e^{x}(1+x)}{\sin^2(xe^x)}dx = \int \frac{1}{\sin^2 t} dt = \int \operatorname{cosec}^2 t \, dt$$

D

$$= -\cot t + c$$

$$= -\cot(xe^x) + c$$

$xe^x = t$

$(1 \cdot e^x + x e^x) dx = dt$

$e^x (1+x) dx = dt$

9

$$\int \frac{dx}{25+16x^2} = \int \frac{1}{(4x)^2+5^2}\,dx$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

$$= \frac{1}{4\cdot 5}\tan^{-1}\left(\frac{4x}{5}\right)+c$$

$$= \frac{1}{20}\tan^{-1}\left(\frac{4x}{5}\right)+c$$

10

$$\int \frac{dx}{\sqrt{9-(2x-3)^2}} = \int \frac{1}{\sqrt{3^2-(2x-3)^2}}\,dx \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}+c$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{2x-3}{3}\right)+c$$

11

$$\int \frac{dx}{\sqrt{(4x-1)^2+25}} = \int \frac{1}{\sqrt{(4x-1)^2+5^2}}\,dx \qquad \int \frac{1}{\sqrt{x^2+a^2}}\,dx$$

$$= \frac{1}{4}\log\left|4x-1+\sqrt{(4x-1)^2+5^2}\right|+c \qquad = \log\left|x+\sqrt{x^2+a^2}\right|+c$$

12

$\frac{1}{(x)^2+a^2}$

$$\int \frac{e^x}{4+e^{2x}}dx$$

$$\int \frac{e^x}{(e^x)^2+2^2}dx$$

D

$e^x = t$

$e^x dx = dt$

$$\int \frac{1}{t^2+2^2}\,dt$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c$$

Q $\int \frac{x}{x^4+1}dx$

$$\int \frac{x}{(x^2)^2+1^2}dx$$

D

$x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$$\int \frac{1}{t^2+1^2}\,\frac{dt}{2}$$

$$= \frac{1}{2}\,\frac{1}{1}\tan^{-1}\left(\frac{t}{1}\right)+c$$

$2x$

Q $\int \frac{x^5}{x^{12}+1}dx$

$$\int \frac{x^5}{(x^6)^2+1^2}$$

D

$x^6 = t$

Q $\int \frac{x}{x^2+1}dx$

$$= \frac{1}{2}\int \frac{2x}{x^2+1}dx$$

$$= \frac{1}{2}\log(x^2+1)+c$$

$$* \int \underline{u}\,\underline{v}\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

I L A T E

- I → $\sin^{-1}$
- L → $\log$
- A → x, 1, $\sqrt{x}$, x^2+1
- T → $\sin$
- E → e^x, 2^x

$$* \int \underbrace{\log x}\cdot\underbrace{1}\,dx = x\log x - x + c$$

Q $\int \underbrace{x}_{u}\cdot\underbrace{\sin x}_{v}\,dx$

$$= x(-\cos x) - \int (1)(-\cos x)\,dx$$

$$= -x\cos x + \sin x + c$$

Q $\int \underbrace{x}_{u}\cdot\underbrace{e^{4x}}_{v}\,dx$

$$= x\frac{e^{4x}}{4} - \int (1)\frac{e^{4x}}{4}\,dx$$

$$= x\frac{e^{4x}}{4} - \frac{1}{4}\frac{e^{4x}}{4} + c$$

$$= \frac{e^{4x}}{4}\left(\frac{x}{1} - \frac{1}{4}\right) + c$$

$$\frac{e^{4x}}{4}\left(\frac{4x-1}{4}\right) + c$$

Q $\int \underbrace{x}_{v}\cdot\underbrace{\log x}_{u}\,dx$

$$= \log x\cdot\frac{x^2}{2} - \int\left(\frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\right)dx$$

$$= \log x\cdot\frac{x^2}{2} - \frac{1}{2}\int x\,dx$$

$$= \log x\cdot\frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2} + c$$

13

$$\int e^{x}\,(\overset{f(x)}{\sin x} + \overset{f'(x)}{\cos x})\,dx$$

$$= e^{x} \sin x + c$$

t

x must $\left\{ \begin{matrix} -x \\ 2x \\ x^2 \end{matrix} \right\} \times$

$$\int e^{x} \left(f(x) + f'(x) \right) dx$$

$$= e^{x} f(x) + c$$

14

$$\int e^{x}\frac{(2x+1)}{2\sqrt{x}}dx$$

$$\int e^{x}\left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x}}+\frac{1}{2\sqrt{x}}\right)dx$$

$$\int e^{x}\left(\frac{\sqrt{x}\,\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}+\frac{1}{2\sqrt{x}}\right)dx$$

$$= e^{x}\sqrt{x}+c$$

15

$$\int \frac{xe^{x}}{(1+x)^2}\,dx = \int e^{x} \frac{x}{(x+1)^2}\,dx$$

$$= \int e^{x}\left(\frac{(x+1)^{1}}{(x+1)^2} - \frac{1}{(x+1)^2}\right)dx$$

$$= \int e^{x}\left((x+1)^{-1} + \frac{(-1)}{(x+1)^2}\right)dx$$

$$= e^{x}(x+1)^{-1}$$

$$= e^{x}\frac{1}{x+1} + C$$

16

$$\int e^x[\underbrace{\ell n(\sec x+\tan x)}+\underbrace{\sec x}]dx$$

$$= e^x \ln(\sec x + \tan x) + c$$

$$\ln(\sec x + \tan x)$$

$$\frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$$

$$\frac{\sec x\,\cancel{(\sec x + \tan x)}}{\cancel{(\sec x + \tan x)}}$$

17

$$\int e^{x}(\tan x-\ln\cos x)\,dx$$

$$\int e^{x}\left(\underbrace{-\ln\cos x}+\tan x\right)dx$$

$$=-e^{x}\ln(\cos x)+c$$

$$=e^{x}\left((-1)\ln(\cos x)\right)+c$$

18

$$\int \frac{e^x(x-1)}{(x+1)^3}dx = \int e^x \frac{(x-1)}{(x+1)^3} dx$$

$$= \int e^x \left(\frac{(x+1)-1-1}{(x+1)^3}\right) dx$$

$$= \int e^x \left(\frac{(x+1)^1}{(x+1)^3} - \frac{2}{(x+1)^3}\right) dx$$

$$= \int e^x \left((x+1)^{-2} + \frac{(-2)}{(x+1)^3}\right) dx = e^x \cdot (x+1)^{-2} + c$$

$$= \frac{e^x}{(x+1)^2} + c$$

19

$$\int e^x(\sin x - \cos x)dx$$

$$\int e^x(\underbrace{-\cos x} + \sin x)\, dx$$

$$= -e^x \cos x + c$$

$x^2 - 2^2 = (x+2)(x-2)$

JAGRUT AWAAZ

20

Tough

Good

$$\int \frac{x^2e^x}{(x+2)^2}dx = \int e^x\left(\frac{(x^2-4)+4}{(x+2)^2}\right)dx$$

$$= \int e^x\left(\frac{\cancel{(x+2)}(x-2)}{(x+2)^{\cancel{2}}} + \frac{4}{(x+2)^2}\right)dx$$

$$= \int e^x\left(\frac{x-2}{x+2} + \frac{4}{(x+2)^2}\right)dx$$

$$= e^x\left(\frac{x-2}{x+2}\right) + C$$

21

$$\int \frac{e^{\tan^{-1} x}(1+x+x^2)}{1+x^2}dx$$

$\tan^{-1}x = t$

22

$\cancel{\ln} x = t \Rightarrow \frac{1}{x}\,dx = dt$

$x = e^t$

$$\int\left(\ln(\ln x) + \frac{1}{\ln x}\right)dx$$

$dx = x\,dt$

$dx = e^t\,dt$

$$\int\left(\ln t + \frac{1}{t}\right)e^t\,dt$$

$= e^t \ln t + c$

$= e^{\ln_e x} \ln(\ln x) + c$

$= x \ln(\ln x) + c$

23

$\ln x = t$

$$\int \frac{\ln x}{(1+\ln x)^2}\, dx$$

$$\int e^t \frac{t}{(t+1)^2}\, dt$$

$$\int e^t \left(\frac{(t+1)'}{(t+1)^2} - \frac{1}{(t+1)^2}\right) dt = \int e^t \left((t+1)^{-1} + \frac{(-1)}{(t+1)^2}\right) dt$$

$$= e^t \cdot (t+1)^{-1} + c$$

$$= \frac{e^t}{t+1} + c = \frac{e^{\log_e x}}{\log x + 1} + c = \frac{x}{\log x + 1} + c$$

24

$$\int \frac{e^x}{x}(1 + x\ell nx)dx = \int e^x \left(\frac{1 + x\ln x}{x} \right) dx$$

$$= \int e^x \left(\frac{1}{x} + \ln x \right) dx$$

$$= e^x \ln x + c$$